- For a flow, when $R_e = \frac{Ux}{v} < 5 \times 10^5$......... Boundary layer is laminar, and

  When $R_e = \frac{Ux}{v} > 5 \times 10^5$......... Boundary layer is called turbulent.

  Where U = free stream velocity, x = distance from the leading edge, and v = kinematic viscosity of fluid.

- The thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity of the free stream. It is denoted by the symbol δ.
- Displacement thickness $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

  It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.

  Or

  It is an additional "wall thickness" that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation.
- Momentum thickness, $\delta_e = \int_0^\delta \left(1 - \frac{u^2}{U^2}\right) dy$

  Energy thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in K.E of the following fluid on account of boundary layer formation.

- Shape factor $= \frac{\text{displacement thickness}}{\text{momentum thickness}} = \frac{\delta^*}{\theta}$

  For linear distribution shape factor = 3.0
- Von Kaman momentum integral equation is given as

  $$\frac{\tau_0}{\rho U^2} = \frac{d\theta}{dx}$$

  Where, $\theta = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$, and τ_0 = shear stress at surface.

Objective Questions

1. In the boundary layer, the flow is
   (a) Viscous and rotational
   (b) Inviscid and irrotational
   (c) Inviscid and rotational
   (d) Viscous and irrotational
2. The critical value of Reynolds number for transition from laminar to turbulent boundary layer in external flows is taken as:
   (a) 2300
   (b) 4000
   (c) 5×10^5

(d) 3×10^6

3. Velocity defect in boundary layer theory is defined as
   (a) The error in the measurement of velocity at any point in the boundary layer
   (b) The difference between the velocity at any point within the boundary layer and the free stream velocity
   (c) The difference between the velocity at any point within the boundary layer and the velocity nearer the boundary
   (d) The ratio between the velocity at a point in the boundary layer and the free stream velocity

4. (i) Assertion (A): in an ideal fluid, separation from a continuous surface would not occur with a positive pressure gradient.

   Reason (R): the velocity within the boundary layer approaches the inviscid velocity asymptotically.

   (ii) Assertion (A): the thickness of boundary layer cannot be exactly defined.

   Reason (R): the velocity within the boundary layer approaches the inviscid velocity asymptotically.

5. Assertion (A): the thickness of boundary layer is an ever increasing one as its distance from the leading edge of the plate increases.

   Reason (R): in practice, 99% of the depth of the boundary layer is attained within a short distance of the leading edge.

6. For the velocity profile u/u_∞ = , the momentum thickness of a laminar boundary layer on a flat plate at a distance of 1 m from leading edge for air (kinematics viscosity $= 2 \times 10^{-5}$ m^2/s) flowing at a free stream velocity of 2 m/s is given by:
   (a) 3.16 mm
   (b) 2.1 mm
   (c) 3.16 m
   (d) 2.1 m

7. A flat plate 2m × 0.4m is a set parallel to a uniform stream of air (density 1.2 kg/m^3 and viscosity 16 centistokes) with its shorter edges along the flow. The air velocity is 30 km/h. what is the approximate estimated thickness of boundary layer at the downstream end of the plate?
   (a) 1.96 mm
   (b) 4.38 mm
   (c) 13.12 mm
   (d) 9.51 mm

8. How is the displacement thickness in the boundary layer analysis defined?
   (a) The layer in which the loss of energy is maximum

(b) The thickness up to which the velocity approaches 99% of the free stream velocity
(c) The distance measured perpendicular to the boundary by which the stream is displaced on account of formation of boundary layer.
(d) The layer which represents reduction in momentum caused by the boundary layer.

9. The displacement thickness at a section, for an air stream ($\rho = 1.2\ kg/m^3$) moving with a velocity of 10 m/s over flat plate is 0.5mm. what is the loss mass rate of flow of air due to boundary layer formation in kg per meter width of plate per second?
(a) 6×10^{-3}
(b) 6×10^{-5}
(c) 3×10^{-3}
(d) 2×10^{-3}

10. If the velocity distribution in a turbulent boundary layer is given by $\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$ then the ratio of displacement thickness to nominal layer thickness will be
(a) 1.0
(b) 0.6
(c) 0.3
(d) 0.1

11. The velocity distribution in the boundary over the face of a high spillway found to have the following from: $\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/9}$

An a certain section, the free stream velocity u_α was found to be 20m/s and the boundary layer thickness was estimated to be 5cm. the displacement thickness ?

(a) 1.0 cm
(b) 2.0 cm
(c) 4.0 cm
(d) 5.0 cm

12. For linear distribution of velocity in the boundary layer on a flat plate, what is the ratio of displacement thickness ( ) to the boundary layer thickness ( )?
(a) 1/4
(b) 1/3
(c) 1/2
(d) 1/5

13. If U^∞=free stream velocity, u = velocity at y and ( ) = boundary layer thickness, then in a boundary layer flow, the momentum thickness ( )is given by :
(a) $\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$
(b) $\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u^2}{u^2{}_\infty}\right) dy$
(c) $\theta = \int_0^\delta \frac{u^2}{u^2{}_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$

(d) $\theta = \int_0^{\delta}\left(1-\frac{u}{u_\infty}\right)dy$

14. Given that

δ = boundary layer thickness

δ^* = displacement thickness

δ_e = energy thickness

θ = momentum thickness

The shape factor H of a boundary layer is given by

(a) $H = \frac{\delta_e}{\delta}$
(b) $H = \frac{\delta^*}{\theta}$
(c) $H = \frac{\delta}{\theta}$
(d) $H = \frac{\delta}{\delta^*}$

15. The velocity distribution in the boundary layer is given as $u/u_s = y/(\ )$, where u is the velocity at a distance y from the boundary, u_s is the free stream velocity and ( ) is the boundary layer thickness at a certain distance from the leading edge of a plate. The ratio of displacement to momentum thickness is :
(a) 5
(b) 4
(c) 3
(d) 2

16. Which one of the following is the correct relationship between the boundary layer thickness δ, displacement thickness δ^* and the momentum thickness?
(a) $\delta > \delta^* > \theta$
(b) $\delta^* > \theta > \delta$
(c) $\theta > \delta > \delta^*$
(d) $\theta > \delta^* > \delta$

17. For air flow over a flat plate, velocity(U) and boundary layer thickness (δ) can be expressed respectively, as

$$\frac{U}{U_\alpha} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3; \qquad \delta = \frac{4.64x}{\sqrt{Re_x}}$$

If the free stream velocity is 2 m/s, and air has kinematic viscosity of $1.5 \times 10^{-5} m^2/s$ and density of $1.23 \times 10^2 kg/m^3$ then wall shear stress at x = 1m, is

(a) $2.36 \times 10^2 N/m^2$
(b) $43.6 \times 10^{-3} N/m^2$
(c) $4.36 \times 10^{-3} N/m^2$