1. The Schrenk Approximation

The Schrenk method relies on the fact that the distribution of lift across the span of an unswept wing does not differ much from elliptic, even for a highly non-elliptic planform. The process required is therefore to create an elliptic planform over a wing semi-span, and then modify it by considering the wing chord variation along the wing.

The first step is to consider the wing without any washout, just looking at the wing planform. I find this the ideal application for a computer spreadsheet. Once set up, the spreadsheet will be there for all design iterations. What we are about to do is to find the lift coefficient distribution along the span assuming an aircraft lift coefficient of 1.0. Based on these results, we will eventually factor this lift coefficient distribution by whatever aircraft lift coefficient we have to obtain the true lift coefficient, and hence lift distribution.

Method

1. Construct a quarter ellipse with a length equal to the semi-span The height of the ellipse is given by the equation:

   $$Ellipse\ Height = \frac{4S}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2}$$ (At the aircraft centre line $Height = \frac{4S}{\pi b}$)

   where S = wing area, b = wing span, y = distance along span from aircraft centre-line

2. Plot this against span location, y, on a graph, and add to this a plot of the wing chord.

3. Draw a line averaging the two plots.

This will generate a curve showing the parameter cC_{la}, where c is the local wing chord and C_{la} is the local lift coefficient for a global lift coefficient, $C_L = 1.0$.

DESIGN EXAMPLE					
SPAN			b	19.0	FT
WING AREA			S	66.5	SQ FT
1	2	3	4	5	6
STATION	2y/b				
(IN)		(FT)		SCRENK	UNIT
WING TIP		CHORD	ELLIPSE	cCla	Cla
114.0	1.000	2.00	0.000	1.000	0.500
110.0	0.965	2.11	1.170	1.638	0.778
105.0	0.921	2.24	1.735	1.986	0.888
100.0	0.877	2.37	2.140	2.254	0.952
95.0	0.833	2.50	2.463	2.482	0.993
90.0	0.789	2.63	2.735	2.683	1.020
85.0	0.746	2.76	2.970	2.866	1.037
80.0	0.702	2.89	3.175	3.035	1.048
75.0	0.658	3.03	3.356	3.191	1.054
70.0	0.614	3.16	3.517	3.338	1.057
60.0	0.526	3.42	3.789	3.605	1.054
50.0	0.439	3.68	4.005	3.845	1.044
40.0	0.351	3.95	4.173	4.060	1.029
30.0	0.263	4.21	4.299	4.255	1.011
25.0	0.219	4.34	4.348	4.345	1.001
20.0	0.175	4.47	4.387	4.430	0.990
15.0	0.132	4.61	4.418	4.511	0.980
0.0	0.000	5.00	4.456	4.728	0.946
WING ROOT					

Table 1. Schrenk Approximation.

Columns are numbered 1 to 6. The equations used in each column are as follows:

Span	b
Wing Area	S
Wing Chord	c

(1) Choose a number of spanwise locations of interest. For example, in this aircraft, the wing root joint is 15 inches outboard from the centre-line. Hence I have chosen a point at station 15. Apart from these points of interest, you may wan to place a point at each rib, if you have decided where the ribs are going. This will enable you to add discrete loads at each rib later on in the analysis for flap loads, etc. On a fabric covered two spar wing, the loads will be input into the spar at each rib location. The rest of the points have been chosen to produce nice graphs!

(2) = 2 x (1)/(12 x b)

(3) Wing chord at each span location

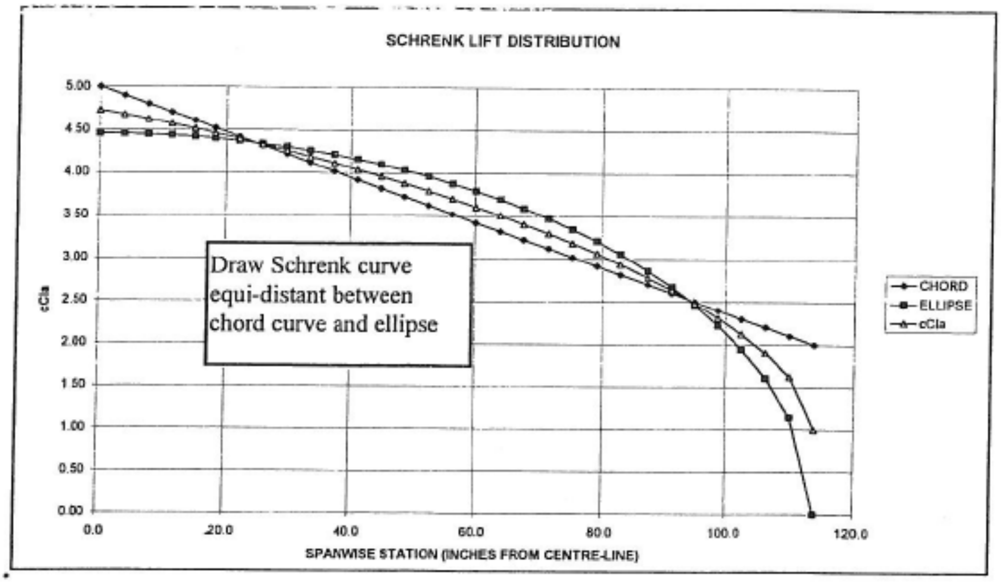
(4) $$Ellipse = \frac{4S}{\pi b}\sqrt{(1-(2)^2)}$$

(5) $$cC_{la} = \frac{(3)+(4)}{2}$$

(6) =(5)/(3)

Note that the odd 12 has crept in to convert inches to feet

From this table, a number of graphs can be plotted. Figure 2 shows the wing chord, the ellipse with height at the aircraft centre line = $\text{Ellipse Height} = \frac{4S}{\pi b}$ and the line drawn between them which gives us local unit lift coefficient multiplied by local chord, cC_{la}. All these terms are plotted against wing semi-span.



Dividing the resulting Schrenk distribution for the untwisted wing, cCl_a, by the local chord, c, will give the local lift coefficient along the span for an aircraft lift coefficient, $C_L = 1.0$, with no washout (see column 6 of table 1).

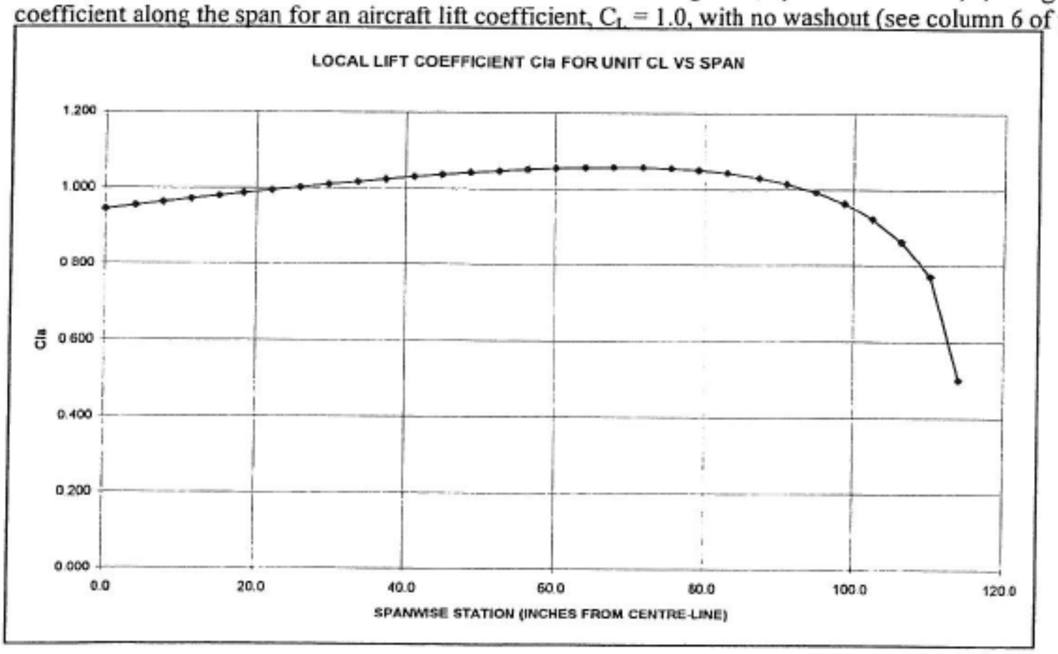


Figure 3